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Comments on "Analysis of an isotropic finite wedge under antiplane deformation" [Int. J. Solids Struct. 34 (1997) 113–128]

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1. Introduction

Kargarnovin et al. (1997) used finite Mellin transforms to obtain the displacement and stress components in an isotropic wedge with finite radius under antiplane deformation (Fig. 1). The boundary condition on the circular segment ($r = a$) may be traction free (Case I) or fixed (Case II). Three different boundary conditions are assigned on the radial edges: traction–displacement (Case a), displacement–displacement (Case b), and traction–traction (Case c). For the cases when traction is applied on the radial edge, they made a conclusion that the stress τ_{rz} and displacement w are divergent at the points of application of tractions. Furthermore, τ_{rz} is discontinuous on the arcs $r = h_1$ and $r = h_2$. This conclusion is incorrect. Obviously, the stress has to be continuous inside the wedge. The objective of this comment is to point out this inconsistency and gives correct results for displacement and stress fields.

2. Incorrect results in Kargarnovin et al. (1997)

The displacements and stress components of Cases Ic and IIc in Kargarnovin et al. (1997) are incorrect. The authors carried out the inverse transform of displacement w by using the residue theory (i.e. Eqs. (34) and (53) in Kargarnovin et al. (1997)):

$$W(r,\theta) = \frac{P}{2\pi i\mu}\int_{c-i\infty}^{c+i\infty} r^{-S}\left[\left(a^{2S}h_2^{-S} + h_2^{S}\right)\cos(S\alpha) - \left(a^{2S}h_1^{-S} + h_1^{S}\right)\right]\frac{\cos(S\theta)}{S\sin(S\alpha)}\,\mathrm{d}S \tag{1}$$

$$W(r,\theta) = -\frac{P}{2\pi i\mu}\int_{c-i\infty}^{c+i\infty} r^{-S}\left[\left(a^{2S}h_2^{-S} - h_2^{S}\right)\cos(S\alpha) - \left(a^{2S}h_1^{-S} - h_1^{S}\right)\right]\frac{\cos(S\theta)}{S\sin(S\alpha)}\,\mathrm{d}S \tag{2}$$

The contour includes two paths: Γ_1 and Γ_2 (Fig. 2). The term $\cos(S\alpha)\cos(S\theta)/\sin(S\alpha)$ will cause the integral diverge along Γ_2. It results in the incorrect expressions of displacements (Eqs. (35), (37) and (54) in

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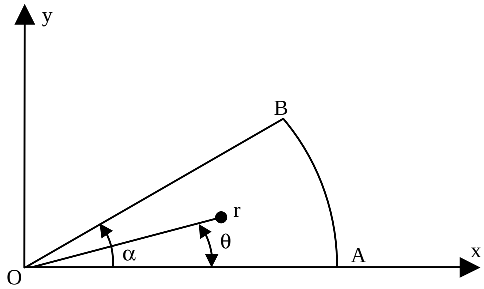

Fig. 1. A finite wedge with radius a and wedge angle α.

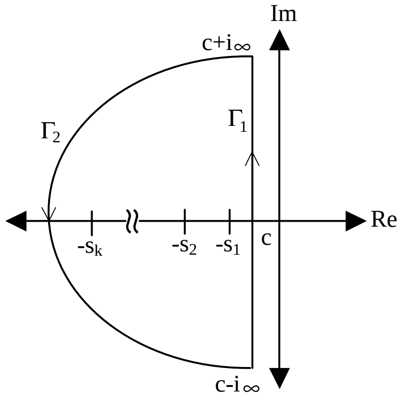

Fig. 2. Integral contours of Γ_1 and Γ_2 in Eqs. (1) and (2).

Kargarnovin et al. (1997)). Consequently, they made an incorrect conclusion that the stress τ_{rz} is discontinuous on the arcs $r = h_1$ and $r = h_2$. In addition, the solution of τ_{rz} in Kargarnovin et al. (1997) has some typo errors.

In addition, same mistake can be found in Cases Ia and IIa in which the traction boundary condition is applied on one radial edge. The stresses τ_{rz} and $\tau_{\theta z}$ at the arcs $r = h_1$ and $r = h_2$ are incorrect and have to be derived separately using a different approach.

3. Our exact solutions

A new coordinate system shown in Fig. 3 is defined. The displacements and the stresses of Cases Ic and IIc are derived analytically. In addition, the stresses for cases Ia and IIa at the arcs $r = h_1$ and $r = h_2$ are also obtained too.

3.1. Case Ic: Traction-free on circular edge r = a and traction–traction on radial edges

The boundary conditions of radial edges become:

$$\tau_{\theta z}\left(r, -\frac{\alpha}{2}\right) = P\delta(r - h_2) \tag{3}$$

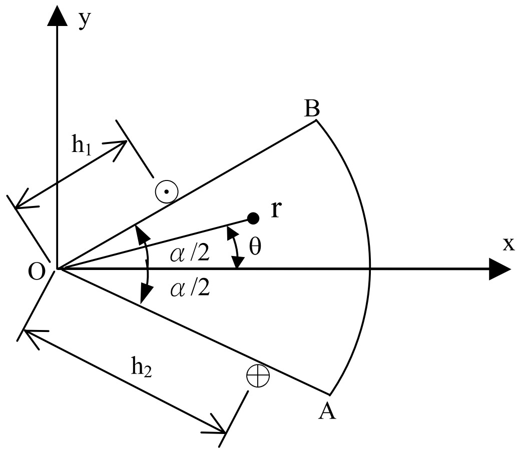

Fig. 3. A finite wedge defined in a new coordinate system.

$$\tau_{\theta z}\left(r,\frac{\alpha}{2}\right) = P\delta(r-h_1) \tag{4}$$

The transformed displacement in this case is obtained as

$$W_2^*(S,\theta) = \frac{P}{\mu}\left[\frac{\left(a^{2S}h_2^{-S}+h_2^S\right)+\left(a^{2S}h_1^{-S}+h_1^S\right)}{2S}\frac{\sin(S\theta)}{\cos\left(\frac{S\alpha}{2}\right)}+\frac{\left(a^{2S}h_2^{-S}+h_2^S\right)-\left(a^{2S}h_1^{-S}+h_1^S\right)}{2S}\frac{\cos(S\theta)}{\sin\left(\frac{S\alpha}{2}\right)}\right] \tag{5}$$

After lengthy mathematical operations, we get the displacements in three different domains $r \leqslant h_1$, $h_1 \leqslant r \leqslant h_2$, and $r \geqslant h_2$:

$$\begin{aligned} W(r,\theta) = \frac{P}{\mu}\Bigg\{ & \sum_{k=0}^{\infty}(-1)^k\frac{1}{(2k+1)\pi}\left(\left(\frac{r}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\left(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}+\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\right)+\left(\frac{r}{h_1}\right)^{\frac{(2k+1)\pi}{\alpha}}+\left(\frac{r}{h_2}\right)^{\frac{(2k+1)\pi}{\alpha}}\right) \\ & \times\sin\left(\frac{(2k+1)\pi\theta}{\alpha}\right)+\sum_{k=1}^{\infty}(-1)^{k+1}\frac{1}{2k\pi}\left(\left(\frac{r}{a}\right)^{\frac{2k\pi}{\alpha}}\left(\left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}}-\left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}\right)\right. \\ & \left.-\left(\frac{r}{h1}\right)^{\frac{2k\pi}{\alpha}}+\left(\frac{r}{h2}\right)^{\frac{2k\pi}{\alpha}}\right)\cos\left(\frac{2k\pi\theta}{\alpha}\right)\Bigg\} \end{aligned} \tag{6a}$$

for $r \leqslant h_1$,

$$\begin{aligned} W(r,\theta) = \frac{P}{\mu}\Bigg\{ & \sum_{k=0}^{\infty}(-1)^k\frac{1}{(2k+1)\pi}\left(\left(\frac{r}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\left(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}+\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\right)+\left(\frac{h_1}{r}\right)^{\frac{(2k+1)\pi}{\alpha}}+\left(\frac{r}{h_2}\right)^{\frac{(2k+1)\pi}{\alpha}}\right) \\ & \times\sin\left(\frac{(2k+1)\pi\theta}{\alpha}\right)+\sum_{k=1}^{\infty}(-1)^{k+1}\frac{1}{2k\pi}\left(\left(\frac{r}{a}\right)^{\frac{2k\pi}{\alpha}}\left(\left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}}-\left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}\right)\right. \\ & \left.-\left(\frac{h_1}{r}\right)^{\frac{2k\pi}{\alpha}}+\left(\frac{r}{h_2}\right)^{\frac{2k\pi}{\alpha}}\right)\cos\left(\frac{2k\pi\theta}{\alpha}\right)+\frac{\ln[h_1]-\ln[r]}{\alpha}\Bigg\} \end{aligned} \tag{6b}$$

for $h_1 \leqslant r \leqslant h_2$, and

$$W(r,\theta) = \frac{P}{\mu}\left\{ \sum_{k=0}^{\infty}(-1)^k \frac{1}{(2k+1)\pi}\left(\left(\frac{r}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\left(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}} + \left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\right) + \left(\frac{h_1}{r}\right)^{\frac{(2k+1)\pi}{\alpha}} + \left(\frac{h_2}{r}\right)^{\frac{(2k+1)\pi}{\alpha}}\right) \times \sin\left(\frac{(2k+1)\pi\theta}{\alpha}\right) + \sum_{k=1}^{\infty}(-1)^{k+1}\frac{1}{2k\pi}\left(\left(\frac{r}{a}\right)^{\frac{2k\pi}{\alpha}}\left(\left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}} - \left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}\right) - \left(\frac{h_1}{r}\right)^{\frac{2k\pi}{\alpha}} + \left(\frac{h_2}{r}\right)^{\frac{2k\pi}{\alpha}}\right)\cos\left(\frac{2k\pi\theta}{\alpha}\right) + \frac{\ln[h_1] - \ln[h_2]}{\alpha}\right\} \tag{6c}$$

for $r \geqslant h_2$. The displacement at wedge apex is set to zero. We will prove numerically that the displacement is continuous within the whole wedge.

The stresses τ_{rz} and $\tau_{\theta z}$ can be obtained in three domains $r < h_1$, $h_1 < r < h_2$, and $r > h_2$ as:

$$\tau_{rz}(r,\theta) = \frac{P}{r\alpha}\left\{ \sum_{k=0}^{\infty}(-1)^k \left(\left(\frac{r}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\left(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}} + \left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\right) + \left(\frac{r}{h_1}\right)^{\frac{(2k+1)\pi}{\alpha}} + \left(\frac{r}{h_2}\right)^{\frac{(2k+1)\pi}{\alpha}}\right) \times \sin\left(\frac{(2k+1)\pi\theta}{\alpha}\right) + \sum_{k=1}^{\infty}(-1)^{k}\left(\left(\frac{r}{a}\right)^{\frac{2k\pi}{\alpha}}\left(\left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}} - \left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}}\right) + \left(\frac{r}{h_1}\right)^{\frac{2k\pi}{\alpha}} - \left(\frac{r}{h_2}\right)^{\frac{2k\pi}{\alpha}}\right)\cos\left(\frac{2k\pi\theta}{\alpha}\right)\right\} \tag{7a}$$

$$\tau_{\theta z}(r,\theta) = \frac{P}{r\alpha}\left\{ \sum_{k=0}^{\infty}(-1)^k \left(\left(\frac{r}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\left(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}} + \left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\right) + \left(\frac{r}{h_1}\right)^{\frac{(2k+1)\pi}{\alpha}} + \left(\frac{r}{h_2}\right)^{\frac{(2k+1)\pi}{\alpha}}\right) \times \cos\left(\frac{(2k+1)\pi\theta}{\alpha}\right) + \sum_{k=1}^{\infty}(-1)^{k}\left(\left(\frac{r}{a}\right)^{\frac{2k\pi}{\alpha}}\left(\left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}} - \left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}\right) - \left(\frac{r}{h_1}\right)^{\frac{2k\pi}{\alpha}} + \left(\frac{r}{h_2}\right)^{\frac{2k\pi}{\alpha}}\right)\sin\left(\frac{2k\pi\theta}{\alpha}\right)\right\} \tag{7b}$$

for $r < h_1$,

$$\tau_{rz}(r,\theta) = \frac{P}{r\alpha}\left\{ \sum_{k=0}^{\infty}(-1)^k \left(\left(\frac{r}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\left(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}} + \left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\right) - \left(\frac{h_1}{r}\right)^{\frac{(2k+1)\pi}{\alpha}} + \left(\frac{r}{h_2}\right)^{\frac{(2k+1)\pi}{\alpha}}\right) \times \sin\left(\frac{(2k+1)\pi\theta}{\alpha}\right) + \sum_{k=1}^{\infty}(-1)^{k}\left(\left(\frac{r}{a}\right)^{\frac{2k\pi}{\alpha}}\left(\left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}} - \left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}}\right) - \left(\frac{h_1}{r}\right)^{\frac{2k\pi}{\alpha}} - \left(\frac{r}{h_2}\right)^{\frac{2k\pi}{\alpha}}\right)\cos\left(\frac{2k\pi\theta}{\alpha}\right) - 1\right\} \tag{7c}$$

$$\tau_{\theta z}(r,\theta)=\frac{P}{r\alpha}\Bigg\{\sum_{k=0}^{\infty}(-1)^k\Bigg(\left(\frac{r}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\Bigg(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}+\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\Bigg)+\left(\frac{h_1}{r}\right)^{\frac{(2k+1)\pi}{\alpha}}+\left(\frac{r}{h_2}\right)^{\frac{(2k+1)\pi}{\alpha}}\Bigg)\times\cos\left(\frac{(2k+1)\pi\theta}{\alpha}\right)+\sum_{k=1}^{\infty}(-1)^k\Bigg(\left(\frac{r}{a}\right)^{\frac{2k\pi}{\alpha}}\Bigg(\left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}}-\left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}\Bigg)-\left(\frac{h_1}{r}\right)^{\frac{2k\pi}{\alpha}}+\left(\frac{r}{h_2}\right)^{\frac{2k\pi}{\alpha}}\Bigg)\sin\left(\frac{2k\pi\theta}{\alpha}\right)\Bigg\} \tag{7d}$$

for $h_1 < r < h_2$, and

$$\tau_{rz}(r,\theta)=\frac{P}{r\alpha}\Bigg\{\sum_{k=0}^{\infty}(-1)^k\Bigg(\left(\frac{r}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\Bigg(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}+\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\Bigg)-\left(\frac{h_1}{r}\right)^{\frac{(2k+1)\pi}{\alpha}}-\left(\frac{h_2}{r}\right)^{\frac{(2k+1)\pi}{\alpha}}\Bigg)\times\sin\left(\frac{(2k+1)\pi\theta}{\alpha}\right)+\sum_{k=1}^{\infty}(-1)^k\Bigg(\left(\frac{r}{a}\right)^{\frac{2k\pi}{\alpha}}\Bigg(\left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}-\left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}}\Bigg)-\left(\frac{h_1}{r}\right)^{\frac{2k\pi}{\alpha}}+\left(\frac{h_2}{r}\right)^{\frac{2k\pi}{\alpha}}\Bigg)\cos\left(\frac{2k\pi\theta}{\alpha}\right)\Bigg\} \tag{7e}$$

$$\tau_{\theta z}(r,\theta)=\frac{P}{r\alpha}\Bigg\{\sum_{k=0}^{\infty}(-1)^k\Bigg(\left(\frac{r}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\Bigg(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}+\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\Bigg)+\left(\frac{h_1}{r}\right)^{\frac{(2k+1)\pi}{\alpha}}+\left(\frac{h_2}{r}\right)^{\frac{(2k+1)\pi}{\alpha}}\Bigg)\times\cos\left(\frac{(2k+1)\pi\theta}{\alpha}\right)+\sum_{k=1}^{\infty}(-1)^k\Bigg(\left(\frac{r}{a}\right)^{\frac{2k\pi}{\alpha}}\Bigg(\left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}}-\left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}\Bigg)-\left(\frac{h_1}{r}\right)^{\frac{2k\pi}{\alpha}}+\left(\frac{h_2}{r}\right)^{\frac{2k\pi}{\alpha}}\Bigg)\sin\left(\frac{2k\pi\theta}{\alpha}\right)\Bigg\} \tag{7f}$$

for $r > h_2$. It can be seen that the singularity order becomes conventional square root singularity when the wedge angle $\alpha = 2\pi$.

The stresses τ_{rz} and $\tau_{\theta z}$ on the circular arcs $r = h_1$ and $r = h_2$ have to be dealt with separately by using the following equations:

$$\tau_{rz}=\mu\frac{\partial W}{\partial r}=\frac{-1}{r}\frac{\mu}{2\pi \mathrm{i}}\int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty}Sr^{-S}W_2^*(S,\theta)\,\mathrm{d}S \tag{8}$$

$$\tau_{\theta z}=\frac{\mu}{r}\frac{\partial W}{\partial\theta}=\frac{\mu}{2\pi \mathrm{i}r}\int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty}r^{-S}\frac{\partial W_2^*(S,\theta)}{\partial\theta}\,\mathrm{d}S \tag{9}$$

Since the deriving procedures are very complicated, we simply write down the results as follows:

$$\tau_{rz}(h_1,\theta)=\frac{-P}{\alpha h_1}\left\{\sum_{k=0}^{\infty}(-1)^{k+1}\left(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}+\left(\frac{h_1}{a}\right)^{\frac{2(2k+1)\pi}{\alpha}}+\left(\frac{h_1}{h_2}\right)^{\frac{(2k+1)\pi}{\alpha}}\right)\sin\left(\frac{(2k+1)\pi\theta}{\alpha}\right)\right.$$
$$\left.+\sum_{k=1}^{\infty}(-1)^{k}\left(\left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}\left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}}-\left(\frac{h_1}{a}\right)^{\frac{4k\pi}{\alpha}}+\left(\frac{h_1}{h_2}\right)^{\frac{2k\pi}{\alpha}}\right)\cos\left(\frac{2k\pi\theta}{\alpha}\right)+\frac{1}{2}\right\} \tag{10a}$$

$$\tau_{\theta z}(h_1,\theta)=\frac{P}{h_1\alpha}\left\{\sum_{k=0}^{\infty}(-1)^{k}\left(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}+\left(\frac{h_1}{a}\right)^{\frac{2(2k+1)\pi}{\alpha}}+\left(\frac{h_1}{h_2}\right)^{\frac{(2k+1)\pi}{\alpha}}\right)\cos\left(\frac{(2k+1)\pi\theta}{\alpha}\right)\right.$$
$$+\sum_{k=1}^{\infty}(-1)^{k}\left(\left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}\left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}}-\left(\frac{h_1}{a}\right)^{\frac{4k\pi}{\alpha}}+\left(\frac{h_1}{h_2}\right)^{\frac{2k\pi}{\alpha}}\right)\sin\left(\frac{2k\pi\theta}{\alpha}\right)$$
$$\left.+\frac{\cos\left(\frac{\pi\theta}{\alpha}\right)}{4\left[1-\sin\left(\frac{\pi\theta}{\alpha}\right)\right]}+\frac{\cos\left(\frac{\pi\theta}{\alpha}\right)}{4\left[1+\sin\left(\frac{\pi\theta}{\alpha}\right)\right]}+\frac{\sin\left(\frac{2\pi\theta}{\alpha}\right)}{2\left[1+\cos\left(\frac{2\pi\theta}{\alpha}\right)\right]}\right\},\quad \theta\neq\pm\frac{\alpha}{2} \tag{10b}$$

for $r=h_1$, and

$$\tau_{rz}(h_2,\theta)=\frac{-P}{\alpha h_2}\left\{\sum_{k=0}^{\infty}(-1)^{k+1}\left(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}+\left(\frac{h_2}{a}\right)^{\frac{2(2k+1)\pi}{\alpha}}-\left(\frac{h_1}{h_2}\right)^{\frac{(2k+1)\pi}{\alpha}}\right)\sin\left(\frac{(2k+1)\pi\theta}{\alpha}\right)\right.$$
$$\left.+\sum_{k=1}^{\infty}(-1)^{k}\left(\left(\frac{h_1}{h_2}\right)^{\frac{2k\pi}{\alpha}}+\left(\frac{h_2}{a}\right)^{\frac{4k\pi}{\alpha}}-\left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}\left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}}\right)\cos\left(\frac{2k\pi\theta}{\alpha}\right)+\frac{1}{2}\right\} \tag{10c}$$

$$\tau_{\theta z}(h_2,\theta)=\frac{P}{h_2\alpha}\left\{\sum_{k=0}^{\infty}(-1)^{k}\left(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}+\left(\frac{h_2}{a}\right)^{\frac{2(2k+1)\pi}{\alpha}}+\left(\frac{h_1}{h_2}\right)^{\frac{(2k+1)\pi}{\alpha}}\right)\cos\left(\frac{(2k+1)\pi\theta}{\alpha}\right)\right.$$
$$+\sum_{k=1}^{\infty}(-1)^{k}\left(\left(\frac{h_2}{a}\right)^{\frac{4k\pi}{\alpha}}-\left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}\left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}}-\left(\frac{h_1}{h_2}\right)^{\frac{2k\pi}{\alpha}}\right)\sin\left(\frac{2k\pi\theta}{\alpha}\right)+\frac{\cos\left(\frac{\pi\theta}{\alpha}\right)}{4\left[1-\sin\left(\frac{\pi\theta}{\alpha}\right)\right]}$$
$$\left.+\frac{\cos\left(\frac{\pi\theta}{\alpha}\right)}{4\left[1+\sin\left(\frac{\pi\theta}{\alpha}\right)\right]}-\frac{\sin\left(\frac{2\pi\theta}{\alpha}\right)}{2\left[1+\cos\left(\frac{2\pi\theta}{\alpha}\right)\right]}\right\},\quad \theta\neq\pm\frac{\alpha}{2} \tag{10d}$$

for $r=h_2$. In deriving the expressions of $\tau_{\theta z}$ in Eqs. (10b) and (10d), θ cannot be equal to $\pm\alpha/2$. From Eqs. (5) and (9), we may see that $\tau_{\theta z}\left(h_1,-\frac{\alpha}{2}\right)$ and $\tau_{\theta z}\left(h_2,\frac{\alpha}{2}\right)$ are zero. In addition, the stress $\tau_{\theta z}$ is singular at the points of applied traction, i.e. $\tau_{\theta z}\left(h_1,\frac{\alpha}{2}\right)\to\infty$ and $\tau_{\theta z}\left(h_2,-\frac{\alpha}{2}\right)\to\infty$. However, the stress τ_{rz} is finite at the points of applied traction. Kargarnovin et al. (1997) predicted that the stress τ_{rz} is divergent on those points.

3.2. Case IIc: Fixed-displacement on circular edge $r = a$ and traction–traction on radial edges

The transformed displacement in this case is:

$$W_1^*(S,\theta) = \frac{P}{\mu}\left[\frac{\left(a^{2S}h_2^{-S} - h_2^S\right) + \left(a^{2S}h_1^{-S} - h_1^S\right)}{2S}\frac{\sin(S\theta)}{\cos\left(\frac{S\alpha}{2}\right)} + \frac{\left(a^{2S}h_2^{-S} - h_2^S\right) - \left(a^{2S}h_1^{-S} - h_1^S\right)}{2S}\frac{\cos(S\theta)}{\sin\left(\frac{S\alpha}{2}\right)}\right] \tag{11}$$

Using similar computational procedures of previous Case Ic, we get the displacements in three different domains $r \leqslant h_1$, $h_1 \leqslant r \leqslant h_2$, and $r \geqslant h_2$:

$$\begin{aligned} W(r,\theta) = \frac{-P}{\mu}\Bigg\{ & \sum_{k=0}^{\infty}(-1)^k\frac{1}{(2k+1)\pi}\left(\left(\frac{r}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\left(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}} + \left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\right) - \left(\frac{r}{h_1}\right)^{\frac{(2k+1)\pi}{\alpha}}\right. \\ & \left. - \left(\frac{r}{h_2}\right)^{\frac{(2k+1)\pi}{\alpha}}\right)\sin\left(\frac{(2k+1)\pi\theta}{\alpha}\right) + \sum_{k=1}^{\infty}(-1)^{k+1}\frac{1}{2k\pi}\left(\left(\frac{r}{a}\right)^{\frac{2k\pi}{\alpha}}\left(\left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}} - \left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}\right)\right. \\ & \left. + \left(\frac{r}{h_1}\right)^{\frac{2k\pi}{\alpha}} - \left(\frac{r}{h_2}\right)^{\frac{2k\pi}{\alpha}}\right)\cos\left(\frac{2k\pi\theta}{\alpha}\right) + \frac{\ln[h_1] - \ln[h_2]}{\alpha}\Bigg\} \end{aligned} \tag{12a}$$

for $r \leqslant h_1$,

$$\begin{aligned} W(r,\theta) = \frac{-P}{\mu}\Bigg\{ & \sum_{k=0}^{\infty}(-1)^k\frac{1}{(2k+1)\pi}\left(\left(\frac{r}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\left(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}} + \left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\right) - \left(\frac{h_1}{r}\right)^{\frac{(2k+1)\pi}{\alpha}}\right. \\ & \left. - \left(\frac{r}{h_2}\right)^{\frac{(2k+1)\pi}{\alpha}}\right)\sin\left(\frac{(2k+1)\pi\theta}{\alpha}\right) + \sum_{k=1}^{\infty}(-1)^{k+1}\frac{1}{2k\pi}\left(\left(\frac{r}{a}\right)^{\frac{2k\pi}{\alpha}}\left(\left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}} - \left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}\right)\right. \\ & \left. + \left(\frac{h_1}{r}\right)^{\frac{2k\pi}{\alpha}} - \left(\frac{r}{h_2}\right)^{\frac{2k\pi}{\alpha}}\right)\cos\left(\frac{2k\pi\theta}{\alpha}\right) + \frac{\ln[r] - \ln[h_2]}{\alpha}\Bigg\} \end{aligned} \tag{12b}$$

for $h_1 \leqslant r \leqslant h_2$, and

$$\begin{aligned} W(r,\theta) = \frac{-P}{\mu}\Bigg\{ & \sum_{k=0}^{\infty}(-1)^k\frac{1}{(2k+1)\pi}\left(\left(\frac{r}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\left(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}} + \left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\right) - \left(\frac{h_1}{r}\right)^{\frac{(2k+1)\pi}{\alpha}} - \left(\frac{h_2}{r}\right)^{\frac{(2k+1)\pi}{\alpha}}\right) \\ & \times\sin\left(\frac{(2k+1)\pi\theta}{\alpha}\right) + \sum_{k=1}^{\infty}(-1)^{k+1}\frac{1}{2k\pi}\left(\left(\frac{r}{a}\right)^{\frac{2k\pi}{\alpha}}\left(\left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}} - \left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}\right)\right. \\ & \left. + \left(\frac{h_1}{r}\right)^{\frac{2k\pi}{\alpha}} - \left(\frac{h_2}{r}\right)^{\frac{2k\pi}{\alpha}}\right)\cos\left(\frac{2k\pi\theta}{\alpha}\right)\Bigg\} \end{aligned} \tag{12c}$$

for $r \geqslant h_2$. The displacement w has been shifted such that it is fixed at the circular edge $r = a$. We will prove numerically that the displacement is continuous within the whole wedge.

The stresses τ_{rz} and $\tau_{\theta z}$ in three domains $r < h_1$, $h_1 < r < h_2$, and $r > h_2$ are:

$$\tau_{rz}(r,\theta) = \frac{-P}{r\alpha}\Bigg\{\sum_{k=0}^{\infty}(-1)^k\Bigg(\left(\frac{r}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\Bigg(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}} + \left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\Bigg) - \left(\frac{r}{h_1}\right)^{\frac{(2k+1)\pi}{\alpha}} - \left(\frac{r}{h_2}\right)^{\frac{(2k+1)\pi}{\alpha}}\Bigg) \times \sin\left(\frac{(2k+1)\pi\theta}{\alpha}\right) + \sum_{k=1}^{\infty}(-1)^k\Bigg(\left(\frac{r}{a}\right)^{\frac{2k\pi}{\alpha}}\Bigg(\left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}} - \left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}}\Bigg) - \left(\frac{r}{h_1}\right)^{\frac{2k\pi}{\alpha}} + \left(\frac{r}{h_2}\right)^{\frac{2k\pi}{\alpha}}\Bigg)\cos\left(\frac{2k\pi\theta}{\alpha}\right)\Bigg\} \tag{13a}$$

$$\tau_{\theta z}(r,\theta) = \frac{-P}{r\alpha}\Bigg\{\sum_{k=0}^{\infty}(-1)^k\Bigg(\left(\frac{r}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\Bigg(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}} + \left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\Bigg) - \left(\frac{r}{h_1}\right)^{\frac{(2k+1)\pi}{\alpha}} - \left(\frac{r}{h_2}\right)^{\frac{(2k+1)\pi}{\alpha}}\Bigg) \times \cos\left(\frac{(2k+1)\pi\theta}{\alpha}\right) + \sum_{k=1}^{\infty}(-1)^k\Bigg(\left(\frac{r}{a}\right)^{\frac{2k\pi}{\alpha}}\Bigg(\left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}} - \left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}\Bigg) + \left(\frac{r}{h_1}\right)^{\frac{2k\pi}{\alpha}} - \left(\frac{r}{h_2}\right)^{\frac{2k\pi}{\alpha}}\Bigg)\sin\left(\frac{2k\pi\theta}{\alpha}\right)\Bigg\} \tag{13b}$$

for $r < h_1$,

$$\tau_{rz}(r,\theta) = \frac{-P}{r\alpha}\Bigg\{\sum_{k=0}^{\infty}(-1)^k\Bigg(\left(\frac{r}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\Bigg(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}} + \left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\Bigg) + \left(\frac{h_1}{r}\right)^{\frac{(2k+1)\pi}{\alpha}} - \left(\frac{r}{h_2}\right)^{\frac{(2k+1)\pi}{\alpha}}\Bigg) \times \sin\left(\frac{(2k+1)\pi\theta}{\alpha}\right) + \sum_{k=1}^{\infty}(-1)^k\Bigg(\left(\frac{r}{a}\right)^{\frac{2k\pi}{\alpha}}\Bigg(\left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}} - \left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}}\Bigg) + \left(\frac{h_1}{r}\right)^{\frac{2k\pi}{\alpha}} + \left(\frac{r}{h_2}\right)^{\frac{2k\pi}{\alpha}}\Bigg)\cos\left(\frac{2k\pi\theta}{\alpha}\right) + 1\Bigg\} \tag{13c}$$

$$\tau_{\theta z}(r,\theta) = \frac{-P}{r\alpha}\Bigg\{\sum_{k=0}^{\infty}(-1)^k\Bigg(\left(\frac{r}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\Bigg(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}} + \left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\Bigg) - \left(\frac{h_1}{r}\right)^{\frac{(2k+1)\pi}{\alpha}} - \left(\frac{r}{h_2}\right)^{\frac{(2k+1)\pi}{\alpha}}\Bigg) \times \cos\left(\frac{(2k+1)\pi\theta}{\alpha}\right) + \sum_{k=1}^{\infty}(-1)^k\Bigg(\left(\frac{r}{a}\right)^{\frac{2k\pi}{\alpha}}\Bigg(\left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}} - \left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}\Bigg) + \left(\frac{h_1}{r}\right)^{\frac{2k\pi}{\alpha}} - \left(\frac{r}{h_2}\right)^{\frac{2k\pi}{\alpha}}\Bigg)\sin\left(\frac{2k\pi\theta}{\alpha}\right)\Bigg\} \tag{13d}$$

for $h_1 < r < h_2$,

$$\tau_{rz}(r,\theta) = \frac{-P}{r\alpha}\left\{\sum_{k=0}^{\infty}(-1)^k\left(\left(\frac{r}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\left(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}+\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\right)+\left(\frac{h_1}{r}\right)^{\frac{(2k+1)\pi}{\alpha}}+\left(\frac{h_2}{r}\right)^{\frac{(2k+1)\pi}{\alpha}}\right)\right.$$
$$\times\sin\left(\frac{(2k+1)\pi\theta}{\alpha}\right)+\sum_{k=1}^{\infty}(-1)^k\left(\left(\frac{r}{a}\right)^{\frac{2k\pi}{\alpha}}\left(\left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}-\left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}}\right)+\left(\frac{h_1}{r}\right)^{\frac{2k\pi}{\alpha}}\right.$$
$$\left.\left.-\left(\frac{h_2}{r}\right)^{\frac{2k\pi}{\alpha}}\right)\cos\left(\frac{2k\pi\theta}{\alpha}\right)\right\} \tag{13e}$$

$$\tau_{\theta z}(r,\theta) = \frac{-P}{r\alpha}\left\{\sum_{k=0}^{\infty}(-1)^k\left(\left(\frac{r}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\left(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}+\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\right)-\left(\frac{h_1}{r}\right)^{\frac{(2k+1)\pi}{\alpha}}-\left(\frac{h_2}{r}\right)^{\frac{(2k+1)\pi}{\alpha}}\right)\right.$$
$$\times\cos\left(\frac{(2k+1)\pi\theta}{\alpha}\right)+\sum_{k=1}^{\infty}(-1)^k\left(\left(\frac{r}{a}\right)^{\frac{2k\pi}{\alpha}}\left(\left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}}-\left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}\right)+\left(\frac{h_1}{r}\right)^{\frac{2k\pi}{\alpha}}\right.$$
$$\left.\left.-\left(\frac{h_2}{r}\right)^{\frac{2k\pi}{\alpha}}\right)\sin\left(\frac{2k\pi\theta}{\alpha}\right)\right\} \tag{13f}$$

for $r > h_2$.

The stresses on the arcs $r = h_1$ and $r = h_2$ are obtained by using the following relations:

$$\tau_{rz} = \mu\frac{\partial W}{\partial r} = \frac{1}{r}\frac{\mu}{2\pi i}\int_{c-i\infty}^{c+i\infty} Sr^{-S}W_1^*(S,\theta)\,dS \tag{14}$$

$$\tau_{\theta z} = \frac{\mu}{r}\frac{\partial W}{\partial\theta} = \frac{-\mu}{2\pi i r}\int_{c-i\infty}^{c+i\infty} r^{-S}\frac{\partial W_1^*(S,\theta)}{\partial\theta}\,dS \tag{15}$$

The results are:

$$\tau_{rz}(h_1,\theta) = \frac{P}{\alpha h_1}\left\{\sum_{k=0}^{\infty}(-1)^{k+1}\left(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}+\left(\frac{h_1}{a}\right)^{\frac{2(2k+1)\pi}{\alpha}}-\left(\frac{h_1}{h_2}\right)^{\frac{(2k+1)\pi}{\alpha}}\right)\sin\left(\frac{(2k+1)\pi\theta}{\alpha}\right)\right.$$
$$\left.+\sum_{k=1}^{\infty}(-1)^k\left(\left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}\left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}}-\left(\frac{h_1}{a}\right)^{\frac{4k\pi}{\alpha}}-\left(\frac{h_1}{h_2}\right)^{\frac{2k\pi}{\alpha}}\right)\cos\left(\frac{2k\pi\theta}{\alpha}\right)-\frac{1}{2}\right\} \tag{16a}$$

$$\tau_{\theta z}(h_1,\theta) = \frac{-P}{h_1\alpha}\left\{\sum_{k=0}^{\infty}(-1)^k\left(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}+\left(\frac{h_1}{a}\right)^{\frac{2(2k+1)\pi}{\alpha}}-\left(\frac{h_1}{h_2}\right)^{\frac{(2k+1)\pi}{\alpha}}\right)\cos\left(\frac{(2k+1)\pi\theta}{\alpha}\right)\right.$$
$$+\sum_{k=1}^{\infty}(-1)^k\left(\left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}\left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}}-\left(\frac{h_1}{a}\right)^{\frac{4k\pi}{\alpha}}-\left(\frac{h_1}{h_2}\right)^{\frac{2k\pi}{\alpha}}\right)\sin\left(\frac{2k\pi\theta}{\alpha}\right)-\frac{\cos\left(\frac{\pi\theta}{\alpha}\right)}{4\left[1-\sin\left(\frac{\pi\theta}{\alpha}\right)\right]}$$
$$\left.-\frac{\cos\left(\frac{\pi\theta}{\alpha}\right)}{4\left[1+\sin\left(\frac{\pi\theta}{\alpha}\right)\right]}-\frac{\sin\left(\frac{2\pi\theta}{\alpha}\right)}{2\left[1+\cos\left(\frac{2\pi\theta}{\alpha}\right)\right]}\right\},\quad \theta\neq\pm\frac{\alpha}{2} \tag{16b}$$

for $r = h_1$, and

$$\tau_{rz}(h_2,\theta) = \frac{P}{\alpha h_2}\left[\sum_{k=0}^{\infty}(-1)^{k+1}\left(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}} + \left(\frac{h_2}{a}\right)^{\frac{2(2k+1)\pi}{\alpha}} + \left(\frac{h_1}{h_2}\right)^{\frac{(2k+1)\pi}{\alpha}}\right)\sin\left(\frac{(2k+1)\pi\theta}{\alpha}\right) + \sum_{k=1}^{\infty}(-1)^{k}\left(\left(\frac{h_2}{a}\right)^{\frac{4k\pi}{\alpha}} - \left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}\left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}} - \left(\frac{h_1}{h_2}\right)^{\frac{2k\pi}{\alpha}}\right)\cos\left(\frac{2k\pi\theta}{\alpha}\right) - \frac{1}{2}\right] \quad (16c)$$

$$\tau_{\theta z}(h_2,\theta) = \frac{-P}{h_2\alpha}\left\{\sum_{k=0}^{\infty}(-1)^{k}\left(\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{\alpha}} + \left(\frac{h_2}{a}\right)^{\frac{2(2k+1)\pi}{\alpha}} - \left(\frac{h_1}{h_2}\right)^{\frac{(2k+1)\pi}{\alpha}}\right)\cos\left(\frac{(2k+1)\pi\theta}{\alpha}\right) + \sum_{k=1}^{\infty}(-1)^{k}\left(\left(\frac{h_2}{a}\right)^{\frac{4k\pi}{\alpha}} - \left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}\left(\frac{h_2}{a}\right)^{\frac{2k\pi}{\alpha}} + \left(\frac{h_1}{h_2}\right)^{\frac{2k\pi}{\alpha}}\right)\sin\left(\frac{2k\pi\theta}{\alpha}\right) - \frac{\cos\left(\frac{\pi\theta}{\alpha}\right)}{4\left[1-\sin\left(\frac{\pi\theta}{\alpha}\right)\right]} - \frac{\cos\left(\frac{\pi\theta}{\alpha}\right)}{4\left[1+\sin\left(\frac{\pi\theta}{\alpha}\right)\right]} + \frac{\sin\left(\frac{2\pi\theta}{\alpha}\right)}{2\left[1+\cos\left(\frac{2\pi\theta}{\alpha}\right)\right]}\right\}, \quad \theta \neq \pm\frac{\alpha}{2} \quad (16d)$$

for $r = h_2$.

Again, stress $\tau_{\theta z}$ satisfies the following requirements:

$$\begin{aligned} &\tau_{\theta z}\left(h_1,\frac{\alpha}{2}\right) \to \infty \\ &\tau_{\theta z}\left(h_1,-\frac{\alpha}{2}\right) = 0 \\ &\tau_{\theta z}\left(h_2,\frac{\alpha}{2}\right) = 0 \\ &\tau_{\theta z}\left(h_2,-\frac{\alpha}{2}\right) \to \infty \end{aligned} \quad (17)$$

3.3. Case Ia: Traction-free on circular edge $r = a$ and traction–displacement on radial edges

Using the relations (8) and (9) in Case Ia, we get the stresses τ_{rz} and $\tau_{\theta z}$ at $r = h$ as follows:

$$\tau_{rz}(h,\theta) = \frac{-P}{\alpha h}\left\{\sum_{k=0}^{\infty}(-1)^{k+1}\left(\frac{h}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\sin\left(\frac{(2k+1)\pi\theta}{2\alpha}\right)\right\} \quad (18a)$$

$$\tau_{\theta z}(h,\theta) = \frac{P}{\alpha h}\left\{\sum_{k=0}^{\infty}(-1)^{k}\left(\frac{h}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\cos\left(\frac{(2k+1)\pi\theta}{2\alpha}\right) + \frac{\cos\left(\frac{\pi\theta}{2\alpha}\right)}{4\left[1-\sin\left(\frac{\pi\theta}{2\alpha}\right)\right]} + \frac{\cos\left(\frac{\pi\theta}{2\alpha}\right)}{4\left[1+\sin\left(\frac{\pi\theta}{2\alpha}\right)\right]}\right\} \quad (18b)$$

The solution of $\tau_{\theta z}$ satisfies the singular stress requirement at the point of applied traction, i.e. $\tau_{\theta z}(h,\alpha) \to \infty$. We can prove that the stresses in this case are the same as Eq. (7) of Case Ic if we let $h_1 = h_2 = h$ and the wedge angle α of Eq. (7) is replaced by 2α.

3.4. Case IIa: Traction-free on circular edge $r = a$ and traction–displacement on radial edges

Using the relations (14) and (15) in Case IIa, we get the stresses τ_{rz} and $\tau_{\theta z}$ at $r = h$ as follows:

$$\tau_{rz}(h,\theta) = \frac{P}{\alpha h}\left\{\sum_{k=0}^{\infty}(-1)^{k+1}\left(\frac{h}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\sin\left(\frac{(2k+1)\pi\theta}{2\alpha}\right)\right\} \tag{19a}$$

$$\tau_{\theta z}(h,\theta) = \frac{-P}{\alpha h}\left\{\sum_{k=0}^{\infty}(-1)^{k}\left(\frac{h}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\cos\left(\frac{(2k+1)\pi\theta}{2\alpha}\right) - \frac{\cos\left(\frac{\pi\theta}{2\alpha}\right)}{4\left[1-\sin\left(\frac{\pi\theta}{2\alpha}\right)\right]} - \frac{\cos\left(\frac{\pi\theta}{2\alpha}\right)}{4\left[1+\sin\left(\frac{\pi\theta}{2\alpha}\right)\right]}\right\} \tag{19b}$$

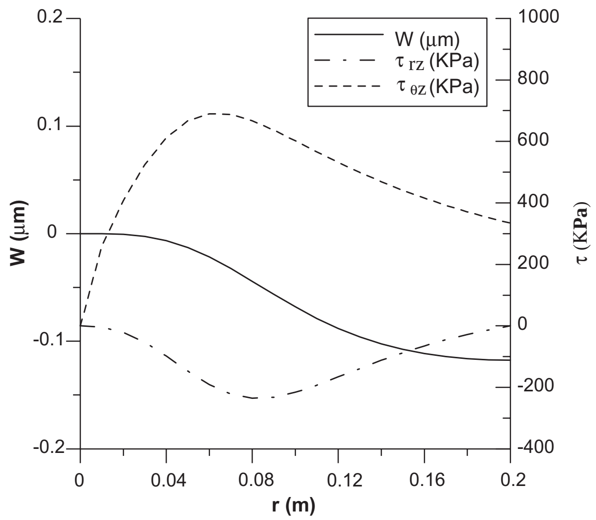

Fig. 4. Variations of w, τ_{rz}, and $\tau_{\theta z}$ with distance r for Case Ic along x-axis ($a = 0.2$ m, $h_1 = 0.08$ m, $h_2 = 0.14$ m, $\alpha = 0.6\pi$).

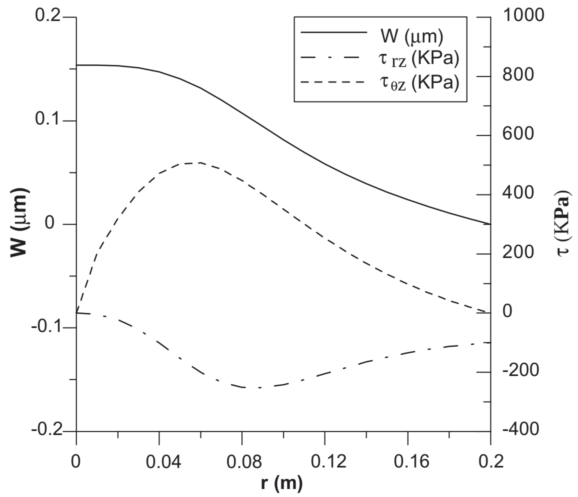

Fig. 5. Variations of w, τ_{rz}, and $\tau_{\theta z}$ with distance r for Case IIc along x-axis ($a = 0.2$ m, $h_1 = 0.08$ m, $h_2 = 0.14$ m, $\alpha = 0.6\pi$).

Again, the solution of $\tau_{\theta z}$ satisfies the singular stress requirement at the point of applied traction, i.e. $\tau_{\theta z}(h, \alpha) \to \infty$.

4. Numerical validation

We will prove that the displacement and stresses are continuous by considering a numerical example: $P = 100$ kN/m, $\mu = 193$ GPa, $a = 0.2$ m, $h_1 = 0.08$ m, $h_2 = 0.14$ m, $\alpha = 0.6\pi$. Figs. 4 and 5 plot the variations of w, τ_{rz}, and $\tau_{\theta z}$ of Cases Ic and IIc along the x-axis (i.e. $\theta = 0°$), respectively. These two plots are accomplished by using 16 000 and 100 terms for displacement and stresses, respectively. All of these quantities are continuous.

Reference

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